THE MEASUREMENT OF THE LONGITUDINAL EMITTANCE OF THE TRANSITION SECTION LU-222

Hua-shun Zhang

4 AUG 1993

The longitudinal emittance is determined by three ellipse parameters, σ_{11}, σ_{12}, σ_{22}, which can be calculated by three equations. There are only two bunch length detectors in the transition section as shown in Fig.1. According to the design data [1], the parameters of the longitudinal emittance at the exit of Tank-5 is : $\epsilon_l = \pi\times\Delta\varphi_o\times\Delta W_o \doteq \pi\times 4.4^{\circ}\times 4\times 412$ (keV $\cdot$deg)=7.24 π(MeV$\cdot$deg)=0.0126 π(MeV$\cdot$rad) =$2.5\times10^{-5}\pi$eV$\cdot$s (in 805 MHz), $\alpha_l = 0$, $\beta_\varphi = 43.2$ (deg/MeV). Thus, $\sigma_{11} = 312.8\,\mathrm{deg}^2 = 0.0928\,\mathrm{rad}^2$, $\sigma_{12} = 0$, $\sigma_{22} = 0.1676\,\mathrm{MeV}^2$. In the space of $(\Delta\varphi, \Delta W)$, the transformation matrix in drift space is :

$$\begin{aligned}\begin{pmatrix}\Delta\varphi_2\\ \Delta W_2\end{pmatrix} &= D\begin{pmatrix}\Delta\varphi_1\\ \Delta W_1\end{pmatrix} = \begin{pmatrix}D_{11} & D_{12}\\ D_{21} & D_{22}\end{pmatrix}\begin{pmatrix}\Delta\varphi_1\\ \Delta W_1\end{pmatrix}\\ &= \begin{pmatrix}1 & \frac{lQ}{\gamma(1+\gamma)W}\\ 0 & 1\end{pmatrix}\begin{pmatrix}\Delta\varphi_1\\ \Delta W_1\end{pmatrix} \qquad (1)\end{aligned}$$

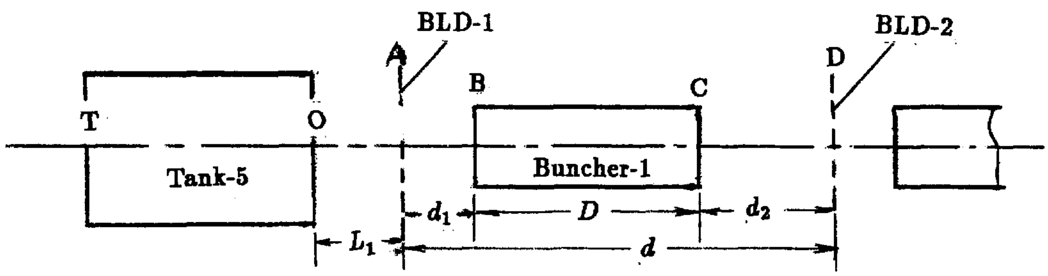

Figure 1: The layout of the transition section

where $Q = \frac{2\pi}{\beta\lambda}$, l is the drift distance. Assume L_1 =0.31 m, the design parameters at the bunch detector-1, point A, are: $\sigma_{11} = 0.09556841$, $\sigma_{12} = 6.895 \times 10^{-3}$, $\sigma_{22} = 0.1676$.

If we will measure the emittance at point A, we get the first equation:

$$\sigma_{11} = \phi_1^2 \tag{2}$$

where ϕ_1 is the measured half beam bunch length (the design value is $\sim 17.7^\circ$). If we use a thin-lens model instead of the actual buncher tank, the transformation matrix from B to C is:

$$\begin{aligned}\begin{pmatrix} \Delta\varphi_c \\ \Delta W_c \end{pmatrix} &= M \begin{pmatrix} \Delta\varphi_b \\ \Delta W_b \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} \Delta\varphi_b \\ \Delta W_b \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ eV_o & 1 \end{pmatrix} \begin{pmatrix} \Delta\varphi_b \\ \Delta W_b \end{pmatrix}\end{aligned} \tag{3}$$

By the design data, after buncher $\Delta W_c \approx 0.82$ MeV. The design value of m_{21} may be estimated by the equation:

$$\Delta W_c^2 = m_{21}^2 \sigma_{11B} + 2m_{21}\sigma_{12B} + \sigma_{22B} \tag{4}$$

The design value $m_{21} \approx$ -2.4387944.

We can use different approaches for the other two equations, e. g.

(1) using a drift equation and a bunch equation: we have

$$\Delta\phi_{2_{off}} = D_{AD_{11}}^2 \sigma_{11A} + 2D_{AD_{11}} D_{AD_{12}} \sigma_{12A} + D_{AD_{12}}^2 \sigma_{22A} \tag{5}$$

$$\Delta\phi_{3_{buncher-on}} = R_{B_{11}}^2 \sigma_{11A} + 2R_{B_{11}} R_{B_{12}} \sigma_{12A} + R_{B_{12}}^2 \sigma_{22A} \tag{6}$$

where $D_{AD_{11}} = 1, D_{AD_{12}} = \frac{dQ}{\gamma(1+\gamma)W}$, $R_B = D_{CD} M_{BC} D_{AB}$ is the assembly matrix from A to D.

(2) using a drift equation (5) and a debunch equation :

$$\Delta\phi_{3_{debuncher-on}} = R_{D_{11}}^2 \sigma_{11A} + 2R_{D_{11}} R_{D_{12}} \sigma_{12A} + R_{D_{12}}^2 \sigma_{22A} \tag{7}$$

where $R_D = D_{CD} M_{BC} D_{AB}$ with $m_{21} > 0$, e. g. $m_{21} = +2.4388$.

(3) using a bunch equation (6) and a debunch equation (7).

The design values of the bunch length in the thin-lens mode are as follows: the phase length at point D at buncher-off $\Delta\phi_{2_{off}} \approx 20.35^\circ$, the phase length at buncher-on $\phi_{3_{buncher-on}} \approx 11.572^\circ$, the phase length at debuncher-on $\Delta\phi_{3_{debuncher-on}} \approx 25.81^\circ$.

(4) using two drift equation (1) from T to A and T to D when Tank-5 is turn-off, and one equation at point A or point D when Tank-5 is turn-on to calculate the emittance at point T, then the emittance at point O can be theoretically calculated and so on.

All the approaches reduces to solve the following equations:

$$
\begin{aligned}
&A\sigma = F \\
&a_{11}\sigma_{11} + a_{12}\sigma_{12} + a_{13}\sigma_{22} = F_1 = \phi_1^2 \\
&a_{21}\sigma_{11} + a_{22}\sigma_{12} + a_{23}\sigma_{22} = F_2 = \phi_2^2 \\
&a_{31}\sigma_{11} + a_{32}\sigma_{12} + a_{33}\sigma_{22} = F_3 = \phi_3^2
\end{aligned} \tag{8}
$$

As known from the theory of linear algebraic equation, the error of the solution of σ, $\Delta\sigma$, caused by the errors of the coefficients of A_{ij} and F_{ij}, ΔA and ΔF, is determined by:

$$
\begin{aligned}
\frac{\| \Delta\sigma \|}{\| \sigma \|} &\leq \frac{\| A \| \cdot \| A^{-1} \|}{1 - \| A^{-1} \| \cdot \| \Delta A \|} \left(\frac{\| \Delta W \|}{\| W \|} + \frac{\| \Delta A \|}{\| A \|} \right) \\
&\approx \| A \| \cdot \| A^{-1} \| \left(\frac{\| \Delta W \|}{\| W \|} + \frac{\| \Delta A \|}{\| A \|} \right)
\end{aligned} \tag{9}
$$

where $\| A \|$ etc. are the norm of matrice A etc. Thus the error is determined by the condition number $COND$ of the matrix A :

$$
COND = \| A \| \cdot \| A^{-1} \| > 1 \tag{10}
$$

For an equation with a large condition number, a small error of the coefficients of A and F will result in a large change in the solution of σ. For the case of first three approaches we have $a_{12} = a_{13} = 0$, it becomes a two dimension equation, so the condition number can be solved as follows :

$$
\begin{aligned}
&COND = \frac{1}{| a_{22}a_{33} - a_{23}a_{32} |} \times \\
&\mathrm{MAX}\{| a_{22} | + | a_{32} |, | a_{23} | + | a_{33} |\} \cdot \mathrm{MAX}\{| a_{32} | + | a_{33} |, | a_{22} | + | a_{23} |\}
\end{aligned} \tag{11}
$$

The condition numbers of the first three approaches are calculated as a function of the buncher rf amplitude and shown in Fig.2. It can be concluded that the condition number is very large, that means a small measurement error in ϕ, $\delta\varphi$, e. g. $\delta\varphi/\phi \sim 1\%$, $\delta\varphi \sim 0.2^\circ$, may cause $\sim 100\%$ error in σ, when $COND \sim 100$. In order to check this conclusion, the σ_{ij} are calculated at different value of ϕ_3, the results are shown in Tab.1.

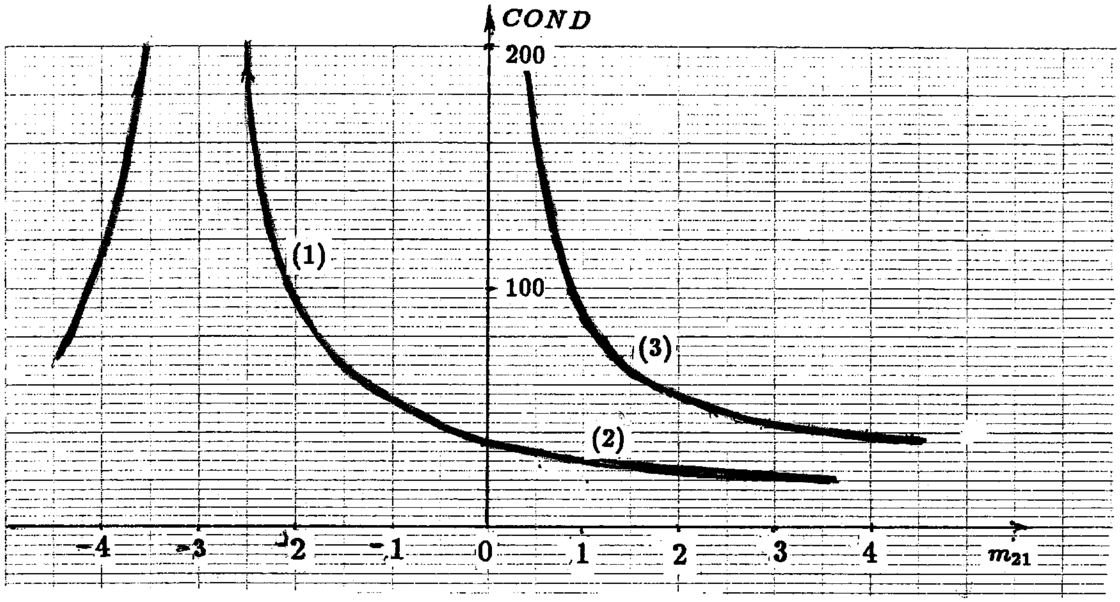

Figure 2: The condition number as a function of rf amplitude

Tab.1 The effect of the value of ϕ_3 on the result

Method	m_{21}	$\phi_1(°)$	$\phi_2(°)$	$\phi_3(°)$	σ_{11}	σ_{12}	σ_{22}	ϵ_l
(1)	-2.439	17.7	20.35	11.572	0.09543	0.007158	0.1673	0.1262
				12.0	—	—	—	imag.num.
				11.0	0.09543	-0.1010	0.7250	0.2429
				10.5	0.09543	-0.1910	1.189	0.2775
(2)	+2.439	17.7	20.35	25.81	0.09543	0.007047	0.1679	0.1264
				26.81	—	—	—	imag.num.
				26.3	0.09543	0.02789	0.06039	0.0706
				24.81	0.09543	-0.03429	0.3810	0.1876
(3)	±2.439	17.7	11.565	25.8	0.09544	0.006748	0.1685	0.1266
				26.8	—	—	—	imag.num.
				26.3	0.09544	0.03126	0.0177	0.02669
				24.8	0.09544	-0.04087	0.4614	0.2058

As shown in Tab.1, an error of 1.0° may cause a unresolved equation. Therefore, the accuracy of this method is questionable.

One better way to solve this problem is to measure the beam energy divergency, $\Delta W_{1/2}$. If it is possible, we have :

$$\sigma_{11} = \phi_1^2 \tag{12}$$

$$\sigma_{22} = \Delta W_{1/2}^2 \tag{13}$$

$$\sigma_{12} = \frac{\phi_2^2 - \phi_1^2 - \left(\frac{dQ}{\gamma(1+\gamma)W}\right)^2 \Delta W_{1/2}^2}{\frac{2dQ}{\gamma(1+\gamma)W}} \tag{14}$$

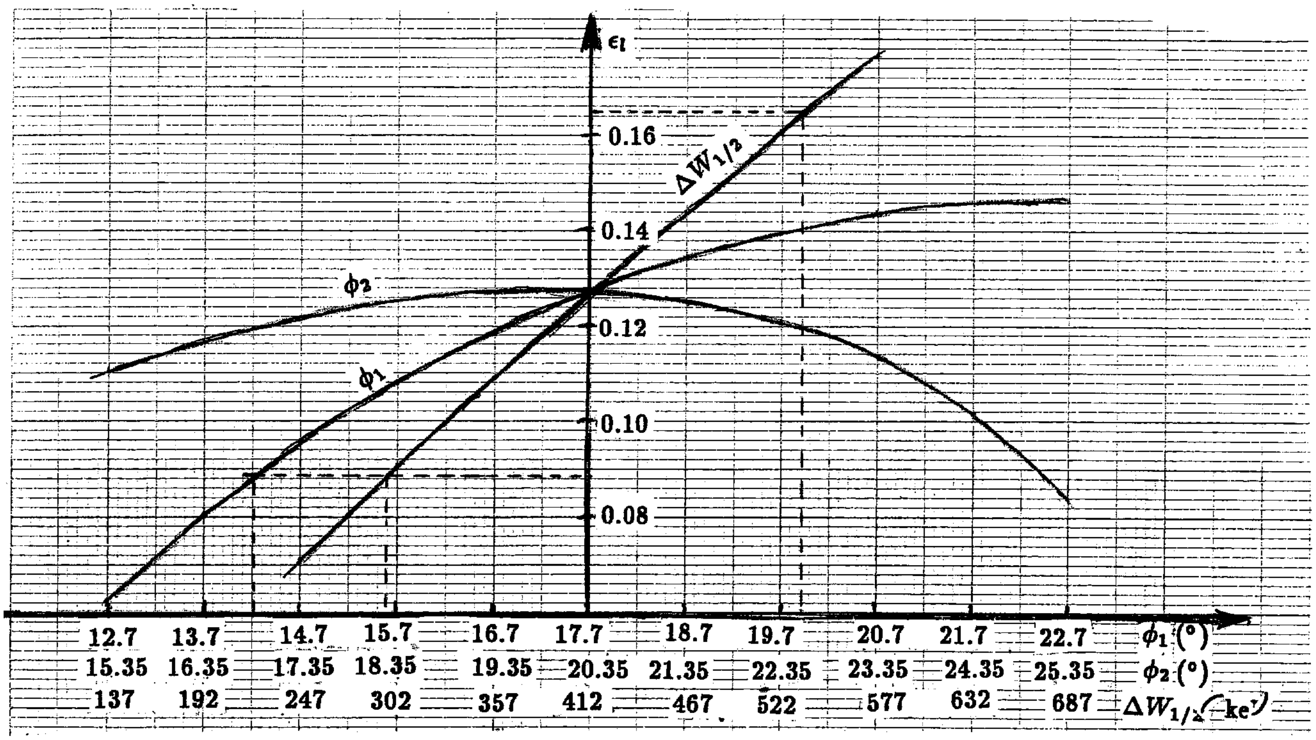

Figure 3: The effect of measurement error on the calculated emittance.

$$\epsilon_l = \sqrt{\sigma_{11}\sigma_{22} - \sigma_{12}^2} \tag{15}$$

The resulted errors of the emittance by an error of singular factor of ϕ_1, ϕ_2 or $\Delta W_{1/2}$ are shown in Fig.3. As shown, for the error of emittance of $\pm 30\%$, the requisite accuracy is: $\delta\varphi \approx 7°$, $\Delta W/W \approx 0.2\%$. It may be acceptable.

By the way if we only use two phase length in drift space, we may know the possible maximum emittance region is the parallelogram as shown in Fig.4, which area is:

$$\epsilon_{max} = \left[\left(\frac{\Delta p}{p}\right)_{pmax} + \left(\frac{\Delta p}{p}\right)_{pmin}\right] \times 2\Delta z_1 = 4\Delta z_1 \frac{\Delta z_2}{L}\gamma^2 \tag{16}$$

CONCLUDION

(1) Theoretically, using two buncher detectors to measure the longitudinal emittance is possible, and the equation is complete. However, the actual error

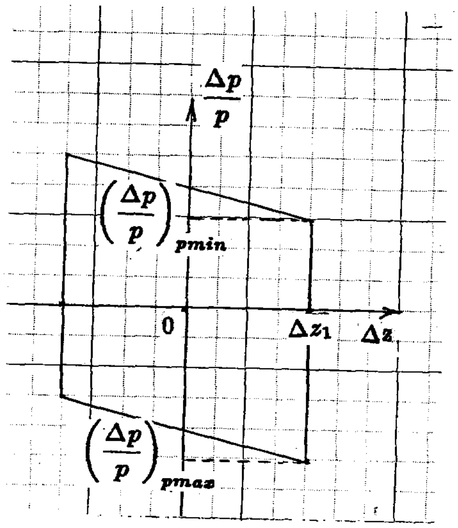

Figure 4: The possible maximum emittance region limited by two phase length.

may be very severe in our case. In order to check the accuracy of the solution, it is suggested to calculate the value of condition number and to use some sets of values of beam lengths caused by possible measurement error to investigate if the solution is stable.

(2) A better approach is to combine the beam length measurement with the measurement of beam energy divergency.

REFERENCE

[1] James A. MacLachlan, LU-158.